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DOES "$\tau \approx 1$" TERMINATE THE ISOTHERMAL EVOLUTION OF COLLAPSING CLOUDS?

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ABSTRACT

We examine when gravitationally collapsing clouds terminate their isothermal evolution. According to our previous work, the condition with which isothermality is broken down is classified into three cases, i.e., when (1) the compressional heating rate overtakes the thermal cooling rate, (2) the optical depth for thermal radiation reaches unity, or (3) the compressional heating rate becomes comparable with the energy transport rate because of radiative diffusion. In the present paper this classification is extended to more general values of the initial cloud temperature $T_{\rm init}$ and opacity κ, and we determine the critical densities with which these conditions are satisfied. For plausible values of $T_{\rm init}$ and κ, we find that the isothermal evolution ceases when case 1 or 3 is satisfied, and case 2 has no significance. We emphasize that the condition of "$\tau \approx 1$" never terminates isothermality, but nonisothermal evolutions begin either earlier or later depending on the initial temperature and opacity. This result contrasts with the conventional idea that opaqueness breaks isothermality. On the basis of the critical density discussed above, the minimum Jeans mass for fragmentation, M_F, is reconsidered. In contrast to the results by previous authors that M_F is insensitive to $T_{\rm init}$ and κ, we find that M_F can be substantially larger than the typical value of $\sim 10^{-2}\ M_\odot$ depending on $T_{\rm init}$ and κ. In particular, M_F increases with decreasing metallicity, $M_F \propto \kappa^{-1}$, for low-metal clouds. A cloud with $\kappa = 10^{-4}$ cm^2 g^{-1} and $T_{\rm init} = 10$ K yields $M_F = 3.7\ M_\odot$. Finally, our critical densities would be helpful for hydrodynamic simulations that are intended to simply handle the hardening of the equation of state.

Subject headings: hydrodynamics — ISM: clouds — radiative transfer — stars: formation

1. INTRODUCTION

The assumption that the early stage of protostellar collapse proceeds isothermally is widely accepted in studies of star formation. This isothermality is justified because the thermal emission by dust grains is quite effective in the early evolution, and the released gravitational energy can be immediately radiated away. Isothermal evolution, however, ceases when thermal radiation no longer cools the cloud against the compressional heating. Detailed studies of the thermal processes that violate isothermality involve complicated problems including radiative transfer, so quantitative investigations have not been done to date.

On the other hand, recent progress in computational facilities has enabled us to handle such complicated problems rather easily. In our previous work (Masunaga, Miyama, & Inutsuka 1998, hereafter MMI), numerical calculations for protostellar collapse were carried out with an exact treatment for radiative transfer. MMI showed that the condition with which isothermality is violated is classified into three different criteria, as is reviewed in § 2 below. MMI found, by both numerical results and analytical estimates, that small differences in the cloud temperature and opacity cause drastic changes in the critical density, $\rho_{\rm crit}$, which is the central density of the collapsing cloud when the isothermal evolution is terminated.

We generalize the analysis of MMI in the present paper and reach a conclusion that the condition of "$\tau \approx 1$" never terminates isothermality in possible ranges of parameters for actual molecular clouds. This result contrasts with the familiar idea that isothermality is violated when the cloud becomes opaque to its thermal radiation at $\rho_{\rm crit} \sim 10^{-13}$ g cm^{-3}. It is a great necessity to critically reexamine this commonly believed idea, which has been often stated in the literature (Larson 1969; Appenzellar & Tscharnuter 1974; Winkler & Newman 1980).

In the context of star formation, it is important to determine when isothermal evolution ceases. In the spherical collapse of a preprotostellar core, the violation of isothermality leads to the formation of a central adiabatic core (the so-called first core) whose size and mass are characterized by $\rho_{\rm crit}$ (MMI). This critical density has more direct importance in cylindrical collapse, because an isothermally collapsing filament is expected to fragment because of the hardening of the equation of state (Inutsuka & Miyama 1992, 1997, hereafter IM92 and IM97, respectively). IM97 derived $\rho_{\rm crit}$ and the mass of a fragmented core as functions of the initial temperature and opacity. An important implication by IM97 is that the minimum Jeans mass for fragmentation depends sensitively on $T_{\rm init}$ and κ, contrary to the conclusions drawn by previous authors (Low & Lynden-Bell 1976; Rees 1976; Silk 1977; Boss 1988). We will revisit this topic in more detail in § 3.

Multidimensional numerical calculations are commonly carried out under the assumption of an isothermal (or polytropic) equation of state. This is because the radiative transfer equation requires an unacceptable computational effort to solve exactly in multiple dimensions. Some numerical studies have shown that an isothermally collapsing cloud in some cases yields a barred structure and/or fragments in the central region (Truelove et al. 1998 and references therein). These results, however, may depend on

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whether the isothermal approximation remains valid or not throughout the evolution, especially in the central region, where the density can be very high. Furthermore, Truelove et al. (1997, 1998) pointed out that artificial fragments are observed in hydrodynamic simulations when cell size is insufficient to resolve the local Jeans length, which implies that one should also consider the possibility that *physical* fragmentation may not actually occur before isothermality is broken down. Our criteria for the violation of isothermality are useful for checking validity of the isothermal approximation. Moreover, it is possible to harden the equation of state (EOS) when the density reaches a certain critical value in numerical simulations for collapsing clouds (e.g., Burkert, Bate, & Bodenheimer 1997). Our results would be helpful to such calculations, in which the isothermal EOS is employed where $\rho < \rho_{\rm crit}$ and, a harder EOS takes place elsewhere.

In § 2 we review $\rho_{\rm crit}$ as defined by MMI and apply it to more general values of the cloud temperature and opacity. In § 3 the minimum Jeans mass for fragmentation is reexamined on the basis of $\rho_{\rm crit}$ obtained in § 2. Our summary and future tasks are described in § 4.

2. CRITICAL DENSITIES FOR THE VIOLATION OF ISOTHERMALITY

In general, molecular clouds are under thermal balance described by the following relation,

$$\Gamma_g + \Gamma_{\rm ext} = \Lambda_{\rm th} , \tag{1}$$

where Γ_g, $\Gamma_{\rm ext}$, and $\Lambda_{\rm th}$ are the compressional heating rate of gas, the heating rate due to the external sources such as cosmic rays and visual or UV photons from surrounding stars, and the radiative cooling rate, respectively. The isothermal collapse proceeds during $\Gamma_g \ll \Lambda_{\rm th}$ and thus $\Gamma_{\rm ext} \approx \Lambda_{\rm th}$, but isothermality is broken down when the compression of gas becomes so effective as to heat up the cloud sufficiently against the cooling.

In this section we discuss the critical central density $\rho_{\rm crit}$ when isothermal evolution is terminated in gravitational collapse, following the manner developed by IM97 and MMI. MMI found that the condition with which isothermality is broken down is classified into three different criteria, as follows.

1. Isothermality is violated when $\Gamma_g = \Lambda_{\rm th}$ if Γ_g overwhelms $\Lambda_{\rm th}$ before the optical depth of the collapsing cloud core, τ, reaches unity. This critical density is denoted by $\rho_{\rm th}$.
2. For $\tau > 1$, the cooling rate is given by the energy transport rate due to radiative diffusion, $\Lambda_{\rm dif}$, instead of $\Lambda_{\rm th}$. If $\Lambda_{\rm th} > \Gamma_g$ during $\tau < 1$ and $\Lambda_{\rm dif} < \Gamma_g$ when $\tau \approx 1$, the isothermal evolution ceases when τ arrives at unity. The central density when $\tau \approx 1$ is denoted by $\rho_{\tau \sim 1}$.
3. If $\Lambda_{\rm dif} > \Gamma_g$ when $\tau \approx 1$, isothermality survives even after τ exceeds unity until the central density reaches a critical value $\rho_{\rm dif}$, which defines the central density when Γ_g becomes comparable with $\Lambda_{\rm dif}$.

Now we evaluate Γ_g, $\Lambda_{\rm th}$, and $\Lambda_{\rm dif}$ to derive $\rho_{\rm crit}$. Note that the heating and cooling rates are defined per unit mass. For brevity we suppose the local thermodynamical equilibrium (LTE), which admits $\Lambda_{\rm th}$ to be described simply as follows,

$$\Lambda_{\rm th} = 4\kappa(T_{\rm init})\sigma T_{\rm init}^4 , \tag{2}$$

where κ is the frequency-averaged opacity[2] per unit mass, which is independent of density because of LTE, and σ denotes the Stefan-Boltzmann constant. The temperature is kept at $T_{\rm init}$ at the initial isothermal stage. The compressional heating rate for gravitational collapse is (see § 5.2 in MMI)

$$\Gamma_g = A c_s^2 \sqrt{4\pi G\rho} , \tag{3}$$

where c_s is isothermal sonic speed and G is the gravitational constant. A numerical constant A is found to be of order unity and is nearly constant through the evolution (see IM97 and MMI).

The optical depth, τ, of the collapsing cloud is defined by

$$\tau = \int_0^\infty \kappa\rho\, dr . \tag{4}$$

In order to evaluate the optical depth, one must suppose a density structure of the collapsing cloud. For a spherical cloud we employ the isothermal similarity solution obtained by Larson (1969) and Penston (1969). The density distribution in their solution is approximated by $\rho =$ constant for $r < R_b$ and $\rho \propto r^{-2}$ for $r > R_b$, where the boundary radius, R_b, is approximately the Jeans length corresponding to the current central density, ρ_c. Hence R_b is expressed as

$$R_b = C \frac{2\pi c_s}{\sqrt{4\pi G\rho_c}} , \tag{5}$$

where a dimensionless constant C is 0.75, in comparison with numerical results (MMI). Equation (4) is then reduced to

$$\tau \sim \int_0^{R_b} \kappa\rho_c\, dr + \int_{R_b}^\infty \kappa\rho_c \left(\frac{r}{R_b}\right)^{-2} dr = 2\kappa\rho_c R_b . \tag{6}$$

Combining equations (5) and (6),

$$\tau = C\kappa\rho_c \frac{4\pi c_s}{\sqrt{4\pi G\rho_c}} . \tag{7}$$

For a filamentary cloud we use equation (8) following the formulation by IM97, where the isothermal equilibrium configuration is adopted for the density distribution

$$\tau = \frac{\pi}{4} \kappa\rho_c \sqrt{\frac{2c_s^2}{\pi G\rho_c}} \tag{8}$$

(eq. [28] in IM97). This approximation is justified because an isothermal filament in equilibrium can collapse while preserving the initial configuration, i.e., undergo homologous collapse. Moreover, a filament formed by the fragmentation of sheetlike clouds has a line mass only twice as large as the equilibrium line mass (Miyama, Narita, & Hayashi 1987a, 1987b), and therefore the equilibrium structure is

[2] In eq. (2) the frequency-averaged opacity κ should be replaced by the Planck mean opacity. In contrast, the Rosseland mean opacity may be preferred in eqs. (10) and (11) below. We, however, do not discriminate between them for brevity in this paper because the difference in values of these two opacities is small.

applicable also to collapsing clouds. One can see that equations (7) and (8) are in accord with each other except for numerical factors.

The opacity per unit mass is assumed to be constant throughout the collapsing cloud. This assumption is well justified when considering the dust continuum opacity, which is independent of the velocity structure of clouds, in contrast to the atomic or molecular line opacity.

We define the r-parameter, which is the ratio of the radiation energy density to the material internal energy, for $\rho = \rho_{\rm crit}$ as follows:

$$r_{\rm crit} \equiv \frac{E}{\rho e} = \frac{4(\gamma - 1)\sigma T_{\rm init}^4}{c \rho_{\rm crit} c_s^2} . \quad (9)$$

The energy transport rate due to radiative diffusion is (MMI)

$$\Lambda_{\rm dif,\,MD} \sim \frac{E}{\rho t_{\rm dif}} = \frac{4\kappa(T_{\rm init})\sigma T_{\rm init}^4}{\tau^2} , \quad (10)$$

when the internal energy of material dominates the radiation energy density (i.e., $r_{\rm crit} < 1$), and

$$\Lambda_{\rm dif,\,RD} \sim \frac{e}{t_{\rm dif}} = \frac{c_s^2 \kappa(T_{\rm init})\rho c}{(\gamma - 1)\tau^2} , \quad (11)$$

when the radiation energy density dominates (i.e., $r_{\rm crit} > 1$), or collectively

$$\Lambda_{\rm dif} = \min\,(\Lambda_{\rm dif,\,MD}, \Lambda_{\rm dif,\,RD}) , \quad (12)$$

where e and E are the internal energy density of fluid per unit mass and the radiation energy density per unit volume, respectively, and γ denotes ratio of the specific heats (we set $\gamma = 5/3$ in this paper). The radiative diffusion time $t_{\rm dif}$ is defined by $\tau^2 \lambda_p / c$, where $\lambda_p \equiv 1/\kappa\rho$ is the mean free path of a photon and c is the speed of light.

Combining equations (2)–(10), we derive the critical densities for the violation of isothermality. For spherical symmetry,

$$\rho_{\rm th} = 4.7 \times 10^{-15}\ {\rm g\ cm^{-3}} \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)^2 \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{6+2\alpha} , \quad (13)$$

$$\rho_{\tau \sim 1} = 2.6 \times 10^{-13}\ {\rm g\ cm^{-3}} \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)^{-2} \times \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{-1-2\alpha} , \quad (14)$$

and

$$\rho_{\rm dif,\,MD} = 6.9 \times 10^{-14}\ {\rm g\ cm^{-3}} \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)^{-2/3} \times \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{(4-2\alpha)/3} , \quad (15)$$

which correspond to equations (14), (17), and (20) in MMI. Here the opacity is approximated as

$$\kappa(T_{\rm init}) = \kappa_0 \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{\alpha} . \quad (16)$$

Note that in the typical range of temperature for molecular clouds, α in equation (16) coincides β, which is the power-law index in frequency (i.e., $\kappa_\nu \propto \nu^\beta$) for the dust opacity (e.g., Beckwith et al. 1990) and is typically $\sim$1–2 for interstellar dust grains. The normalizing factor, κ_0, reflects metallicity and is $\sim$0.01 cm^2 g^{-1} for nearby molecular clouds.

MMI found that these three critical densities are useful in predicting when the isothermal evolution is terminated, comparing them with radiation hydrodynamic numerical calculations with an exact treatment for radiative transfer in spherical symmetry.

In case of $r_{\rm crit} > 1$, equation (15) above is replaced by the following equation, which is derived using equation (11):

$$\rho_{\rm dif,\,RD} = 1.7 \times 10^{-2}\ {\rm g\ cm^{-3}} \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)^{-2} \times \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{-2-2\alpha} . \quad (17)$$

Note that spherical collapse is decelerated when the equation of state is harder than $\gamma = 4/3$, where γ is the adiabatic component, i.e., $d \ln P / d \ln \rho$. Hence the violation of isothermality does not necessarily cause an immediate influence on the evolution (see MMI). In contrast, cylindrical collapse should be decelerated even by a slight hardening of the isothermal equation of state. Furthermore, IM92 and IM97 have shown that a collapsing filament is expected to fragment if the radial collapse is decelerated. Therefore $\rho_{\rm crit}$ for cylindrical collapse, which is considered below, has more significance than the spherical case.

The critical densities for cylindrical (filamentary) collapse were obtained by IM97 as

$$\rho_{\rm th} = 4.7 \times 10^{-15}\ {\rm g\ cm^{-3}} \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)^2 \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{6+2\alpha} , \quad (18)$$

$$\rho_{\tau \sim 1} = 4.7 \times 10^{-12}\ {\rm g\ cm^{-3}} \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)^{-2} \times \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{-1-2\alpha} . \quad (19)$$

Another critical density, $\rho_{\rm dif}$, which was not considered in IM97, is estimated as

$$\rho_{\rm dif,\,MD} = 4.7 \times 10^{-13}\ {\rm g\ cm^{-3}} \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)^{-2/3} \times \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{(4-2\alpha)/3} , \quad (20)$$

$$\rho_{\rm dif,\,RD} = 5.4\ {\rm g\ cm^{-3}} \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)^{-2} \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{-2-2\alpha} . \quad (21)$$

The difference in $\rho_{\rm crit}$ between spherical and cylindrical cases appears only in numerical factors. In the above equations we assumed that the gas constant $k/\mu m_{\rm H} \equiv c_s^2/T$ is 3.6×10^7 ergs g^{-1} K.

Equations (13)–(15), (17), and (18)–(21) show that the critical densities are determined solely by $T_{\rm init}$ and κ. Thus the criteria for the violation of isothermality can be classified by $T_{\rm init}$ and κ. Figure 1 depicts the critical density (shown by

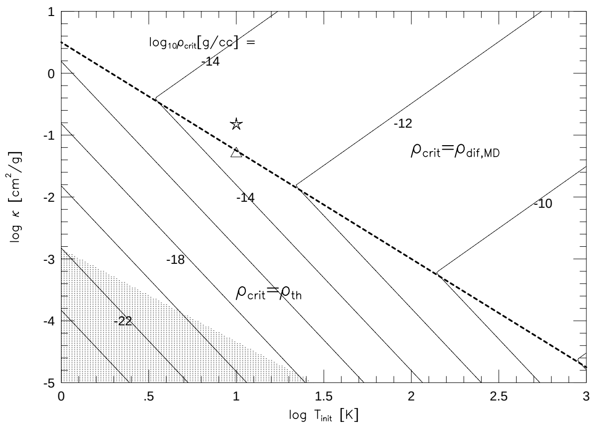

FIG. 1.—Critical density for the violation of isothermality (eqs. [18]–[21]) is shown as contour curves. Note that the vertical axis represents not κ_0 but $\kappa(T_{\rm init})$ (see eq. [16]). Contour levels begin with $\log_{10} \rho_{\rm crit}$ (g cm^{-3}) $= -24$ in the lower left corner and increase as $-22, -20, -18, \ldots$. The dashed line delineates $\rho_{\rm th} = \rho_{\tau\sim 1}$ and, equivalently, $\rho_{\rm dif} = \rho_{\tau \sim 1}$ (see text for detail). The critical density is given by $\rho_{\rm th}$ below the dashed line and by $\rho_{\rm dif}$ above. Shaded domains indicate $r_{\rm crit} > 1$, where $r_{\rm crit}$ is the ratio of the radiation energy density to the material internal energy density and $r_{\rm crit} < 1$ elsewhere. The star and triangle correspond to the model parameters adopted by Larson (1969) and Winkler & Newman (1980), respectively.

contour lines) as a function of $T_{\rm init}$ and κ. We have chosen the range of $T_{\rm init}$ to be less than 1000 K, below which dust grains survive. Although this range is still wider than the possible temperature range for actual molecular clouds, it is helpful to realize the behavior of $\rho_{\rm crit}$ more clearly on the $T_{\rm init}$-κ plane. The vertical axis, κ, is a product of κ_0 and the temperature dependence according to equation (16). Under a given value of $T_{\rm init}$, the vertical axis is an indicator of κ_0 and hence of metallicity.

For $r_{\rm crit} < 1$, boundaries between cases 1 and 3 ($\rho_{\rm th} = \rho_{\tau\sim 1}$ and $\rho_{\rm dif} = \rho_{\tau\sim 1}$) are found to fall onto an equivalent condition, which is delineated by a dashed line in Figure 1. (The reason is described in the next paragraph.) The critical density is given by $\rho_{\rm th}$ below the dashed line and by $\rho_{\rm crit} = \rho_{\rm dif}$ above. The region for $\rho_{\rm crit} = \rho_{\rm dif}$ is divided into two domains: $\rho_{\rm crit} = \rho_{\rm dif,\,RD}$ and $\rho_{\rm crit} = \rho_{\rm dif,\,MD}$. The condition of $\rho_{\rm crit} = \rho_{\rm dif,\,RD}$, however, requires too large a $T_{\rm init}$ and κ compared with the typical values for molecular clouds, and therefore the domain for $\rho_{\rm crit} = \rho_{\rm dif,\,RD}$ does not appear in Figure 1. The shaded domain in the lower left corner, where the isothermality is broken before τ reaches unity, indicates $aT^4 > \rho e$ ($a \equiv 4\sigma/c$), which, however, does not necessarily mean that the radiation energy dominates the fluid internal energy. Unless the external radiation field is intensive enough, E is much less than aT^4 in optically thin media. The model parameters adopted by Larson (1969) and Winkler & Newman (1980) are indicated by the star and triangle, respectively, in Figure 1. Both points read $\rho_{\rm crit} \sim 10^{-13}$ g cm^{-3}, which is in accordance with the central density when the isothermal evolution terminates in their numerical calculations. These points *accidentally* locate near the dashed line, on which the isothermal evolution ceases when $\tau = 1$. This seems to be a major reason why many authors have been misled by the idea that opaqueness terminates isothermality.

Equations (2) and (10) show that $\Lambda_{\rm th}$ and $\Lambda_{\rm dif,\,MD}$ are smoothly connected with each other at $\tau = 1$. In other words, $\rho_{\rm th} = \rho_{\tau\sim 1}$ and $\rho_{\rm dif,\,MD} = \rho_{\tau\sim 1}$ degenerate into an identical line in the $T_{\rm init}$-κ plane (Fig. 1, *dashed line*). Figure 2*a* schematically shows the thermal evolution of a gravitationally collapsing cloud. If $r_{\rm crit} < 1$, the isothermal evolution ceases when Γ_g reaches $\Lambda_{\rm th}$ (which corresponds to case 1 above), or when Γ_g reaches $\Lambda_{\rm dif}$ (case 3). Although neither $\Lambda_{\rm th}$ nor $\Lambda_{\rm dif}$ provides exact estimates for intermediate optical depth, it is clear that the critical density of $\rho_{\tau\sim 1}$ (case 2) plays no essential roles when $r_{\rm crit} < 1$.

In contrast to $\Lambda_{\rm dif,\,MD}$, $\Lambda_{\rm dif,\,RD}$ is not ensured to be connected smoothly with $\Lambda_{\rm th}$ near $\tau \approx 1$, as is found by comparing equations (2) and (11). Therefore, $\rho_{\rm th} = \rho_{\tau\sim 1}$ and $\rho_{\rm dif,\,RD} = \rho_{\tau\sim 1}$ should be indicated by two separate lines in the $T_{\rm init}$-κ plane, and the domain for $\rho_{\rm crit} = \rho_{\tau\sim 1}$ would appear between these two lines. Figure 2*b* draws the thermal evolution for $r_{\rm crit} > 1$, where case 2 appears as well as case 1 and 3. However, $\rho_{\rm dif,\,RD} = \rho_{\tau\sim 1}$ corresponds to $T_{\rm init} = 1.1 \times 10^{13}$ K (!) independently of κ, so case 2 is highly unlikely to occur.

As a conclusion, the critical density for the violation of isothermality is determined by $\rho_{\rm th}$ or $\rho_{\rm dif,\,MD}$, depending on $T_{\rm init}$ and κ, for gravitationally collapsing clouds. *The critical density of $\rho_{\tau\sim 1}$ has no significance in practice.* This fact has been overlooked in all previous works on gravitational collapse of protostellar clouds, including our own papers (IM97 and MMI).

3. THE MINIMUM JEANS MASS FOR FRAGMENTATION

This section is devoted to discussions on the minimum Jeans mass of a fragment of clouds. Previous formulations of the minimum Jeans mass are reconsidered, and we show that they should be modified on the basis of our newly derived criteria for the violation of isothermality.

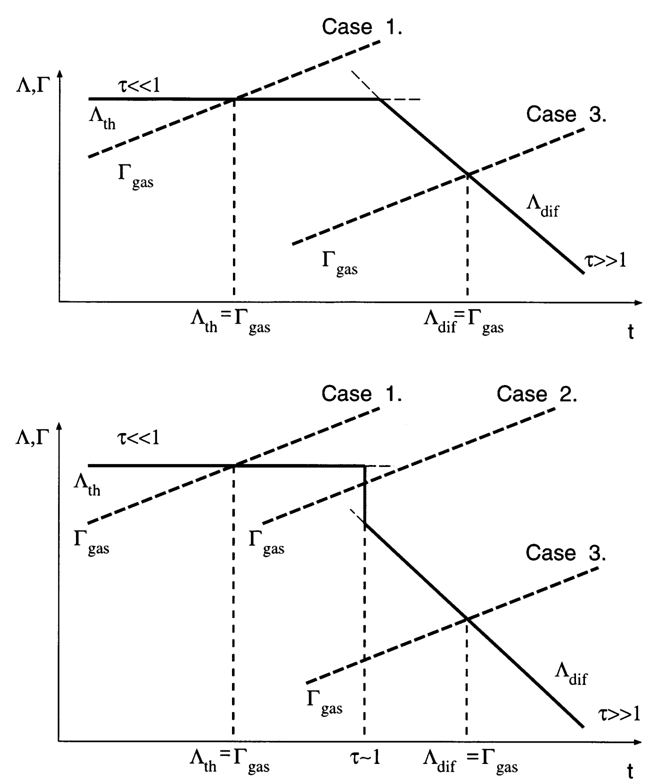

FIG. 2.—Schematic picture illustrating the classification for $\rho_{\rm crit}$ is shown. The heating and cooling rates (Γ_g, $\Lambda_{\rm th}$, and $\Lambda_{\rm dif}$) are depicted as a function of time. The optical depth for thermal radiation, τ, is small enough initially but increases monotonically as the collapse proceeds. The upper panel is for the case of $r_{\rm crit} < 1$ and the lower panel for $r_{\rm crit} > 1$. See text for more detail.

3.1. *Comments on the Previous Studies*

The minimum Jeans mass for fragmentation, M_F, is determined by the condition that the equation of state becomes sufficiently hard to stop further fragmentation. In order to obtain M_F, Low & Lynden-Bell (1976), Silk (1977), and Kanjilal & Basu (1992) imposed two conditions of $\Gamma_g = \Lambda_{\rm th}$ (eq. [3]) and $\tau = 1$ *simultaneously*. As a result, the temperature of clouds that defines M_F is determined uniquely, along with M_F itself. However, as IM97 noted, $T_{\rm init}$ and κ must be left as free parameters, because the cloud temperature is actually given by environmental factors, including the intensity of interstellar radiation and the rate of cosmic-ray heating. Hence we estimate the minimum Jeans mass as a function of both $T_{\rm init}$ and κ on the basis of the discussion on $\rho_{\rm crit}$ in § 2 above. In this manner we propose a more generalized formulation for M_F in § 3.2.

MMI shows that the temperature increases only gradually in case 1 after Γ_g reaches $\Lambda_{\rm th}$, until the temperature eventually begins to rise rapidly when τ exceeds unity. Therefore one might wonder whether the condition of $\Gamma_g = \Lambda_{\rm th}$ can actually trigger the fragmentation of spherical clouds.

Tohline (1980) raised serious objections to the previous models for fragmentation of spherically collapsing clouds. His results indicate that fragmentation occurs after a cloud is collapsed into a highly flattened sheet. A collapsed sheet-like cloud is subject to fragmentation into filaments (Miyama et al. 1987a, 1987b), and therefore the problem of cloud fragmentation becomes a problem of the evolution of filaments. As was mentioned in the previous section, a collapsing filament is expected to fragment by a slight hardening of the isothermal equation of state, in contrast to spherical collapse, so even a gradual increase of temperature can trigger fragmentation (IM92, IM97).

Rees (1976) considered the condition for determining M_F as

$$\frac{GM_F^2}{Rt_{\rm ff}} \sim 4\pi f R^2 \sigma T_{\rm init}^4 \,, \tag{22}$$

where $t_{\rm ff}$ is the free-fall time and f is a nondimensional factor less than unity that depends on the detailed physics of the cooling and opacity. In contrast to the other authors mentioned above, Rees (1976) considered the thermal balance in a collapsing cloud after τ exceeds unity, and thus his criterion (eq. [22]) corresponds to our case 3. This criterion, however, assumes that the released gravitational energy is instantaneously radiated away from the cloud surface. In other words, equation (22) neglects the time required to convey the released energy to the cloud surface by radiative diffusion. However, the energy transport rate due to radiative diffusion is essentially important in case 3, as was described in the previous section. Therefore, the right-hand side of equation (22) should be modified into

$$\int \Lambda_{\rm dif}\, \rho\, dV \sim \frac{4\kappa\sigma T_{\rm init}^4}{\tau^2}\, \rho\pi R^3 \sim \frac{4\pi R^2 \sigma T_{\rm init}^4}{\tau} \,, \tag{23}$$

where we used equation (10) and replaced $\kappa\rho R$ with τ. Equation (23) claims that the right-hand side of equation (22) overestimates the cooling rate by a factor of τ. It may be possible to substitute $1/\tau$ for f in equation (22), but in this case f cannot be a constant factor, since τ varies with ρ (or $t_{\rm ff}$) and R. Furthermore, τ is a function also of both opacity and temperature (because $T_{\rm init}$ characterizes the similarity solution for density structure), and therefore the original formulation by Rees (eq. [22]) requires great care to use.

3.2. *A Revised Definition of the Minimum Jeans Mass*

Following IM97, we evaluate M_F as the mean clump mass of a filament with the effective radius of $H_f = \sqrt{2c_s^2/\pi G\rho_{\rm crit}}$, setting the mean separation between clumps to be $8 \times H_f$:

$$M_F \simeq \rho_{\rm crit}\, \pi H_f^2 (8H_f) = \frac{16c_s^3}{G}\sqrt{\frac{2}{\pi G\rho_{\rm crit}}} \,. \tag{24}$$

Eliminating $\rho_{\rm crit}$ in equation (24) by equations (18) and (20), we have

$$M_{F,\,\rm th} = 3.7 \times 10^{-2}\, M_\odot \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)^{-1} \times \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{-(3+2\alpha)/2} \tag{25}$$

for case 1,

$$M_{F,\,\rm dif,\,MD} = 3.7 \times 10^{-3}\, M_\odot \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)^{1/3} \times \left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{(5+2\alpha)/6} \tag{26}$$

for case 3 with $r_{\rm crit} < 1$, and

$$M_{F,\,\rm dif,\,RD} = 1.1 \times 10^{-9}\, M_\odot \left(\frac{\kappa_0}{0.01\ {\rm cm^2\ g^{-1}}}\right)\left(\frac{T_{\rm init}}{10\ {\rm K}}\right)^{(5+2\alpha)/2} \tag{27}$$

for case 3 with $r_{\rm crit} > 1$.

Figure 3 illustrates M_F by contour lines in the $T_{\rm init}$-κ plane. The dashed line, tracing the bottom of a ravine drawn by contour, represents $M_{F,\,\rm th} = M_{F,\,\tau\sim 1}$ and $M_{F,\,\rm dif,\,MD} = M_{F,\,\tau\sim 1}$ similarly to Figure 1. Then $M_{\rm F}$ equals $M_{F,\,\rm th}$ below the dashed line and $M_F = M_{F,\,\rm dif,\,MD}$ above, according to Figure 1. Recalling that Low & Lynden-Bell (1976) and Silk (1977) considered $\rho_{\rm crit} = \rho_{\rm th}$ and $\rho_{\rm crit} = \rho_{\tau\sim 1}$ simultaneously, one finds that their solution for M_F is constrained on the dashed line in Figure 3. Using equations (25)

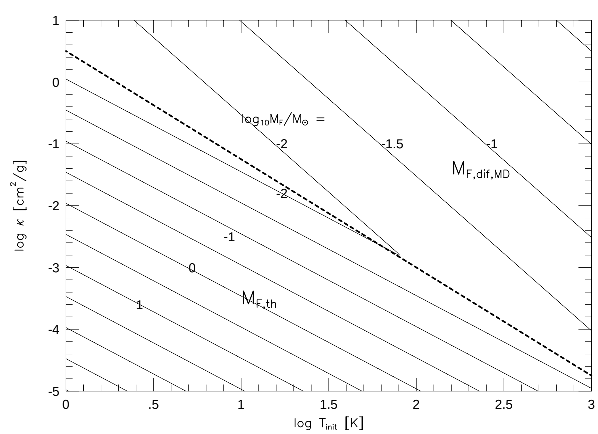

FIG. 3.—Minimum Jeans mass for fragmentation (eqs. [25]–[27]) is shown as contour curves. Note that the vertical axis represents not κ_0 but $\kappa(T_{\rm init})$ (see eq. [16]). Contour levels begin with $\log_{10} M_F/M_\odot = -2$ in the upper left corner and increase as -1.5, -1, -0.5, ... The dashed line delineates $M_{F,\,\rm th} = M_{F,\,\tau\sim 1}$ and, equivalently, $M_{F,\,\rm dif} = M_{F,\,\tau\sim 1}$ according to Fig. 1. The minimum Jeans mass is given by $M_{F,\,\rm th}$ below the dashed line and by $M_{F,\,\rm dif}$ above.

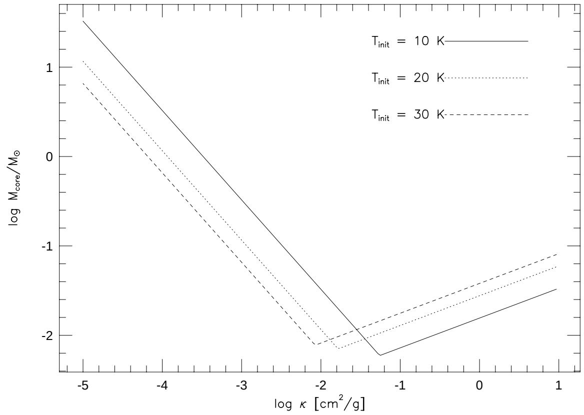

FIG. 4.—Minimum Jeans masses for fragmentation under given temperatures. Solid, dotted, and dashed curves correspond to $T_{init} = 10$, 20, and 30 K, respectively.

and (26), one obtains M_F defined on the dashed line by

$$M_F = 8.4 \times 10^{-3}\, M_{\odot} \left(\frac{\kappa_0}{0.01\ \mathrm{cm^2\ g^{-1}}} \right)^{-1/(7+4\alpha)}, \quad (28)$$

which indeed recovers M_F as defined by Low & Lynden-Bell (1976) and Silk (1977). Low & Lynden-Bell emphasized the weak dependence of M_F on κ_0, as is seen in equation (28).

On the other hand, if assuming that T_{init} does not vary significantly, as is the case in actual molecular clouds, equation (25) shows that the dependence on κ_0 is no longer weak. For the typical temperature of molecular clouds $T_{init} \sim 10$–20 K and for the typical dust opacity $\kappa \simeq 0.01$ cm^2 g^{-1}, Figure 3 shows $M_F = M_{F,\mathrm{th}}$. Hence we choose equation (25), which reads $M_F \simeq 10^{-2}\, M_{\odot}$. Now we consider how M_F varies with the decreasing dust opacity. As was mentioned in § 2, lowering κ under a fixed T_{init} corresponds to lowering metallicity of the clouds. Equation (25) tells us that M_F can be significantly larger than $\sim 10^{-2}\, M_{\odot}$, which is inversely proportional to κ_0 for low-metal clouds. Figure 4 delineates M_F as a function of κ under fixed initial temperatures: $T_{init} = 10$, 20, and 30 K. For instance, a cloud with $T_{init} = 10$ K and $\kappa = 10^{-4}$ cm^2 g^{-1} (one-hundredth as small as the typical metallicity) provides $M_F = 3.7\ M_{\odot}$. This fact might be responsible for the cutoff, if it exists, at the low-mass end of the initial mass function.

4. SUMMARY

Our findings in the present paper are summarized as follows.

1. We derived three criteria for the central density ρ_{crit} when isothermality is violated in gravitationally collapsing clouds. These criteria are supported by numerical results in our previous work. We classified ρ_{crit} in the T_{init}-κ plane and found that ρ_{th} or $\rho_{dif,\,MD}$ determines ρ_{crit} in plausible ranges of parameters for actual molecular clouds. Another criterion, $\rho_{crit} = \rho_{\tau \sim 1}$, actually has no importance.
2. This result indicates that the condition of "$\tau \approx 1$" never violates isothermality. Instead, nonisothermal evolutions begin either earlier or later, depending on the initial temperature and opacity, than the optical depth reaches unity. The conventional idea that the isothermal approximation is valid during the period when the central density is less than 10^{-13} g cm^{-3} is not supported physically.
3. The minimum Jeans mass for fragmentation, M_F, is characterized by ρ_{crit}, hence M_F is a function of both T_{init} and κ as well as ρ_{crit}. The typical value of M_F is $\sim 10^{-2}\, M_{\odot}$ in accordance with the commonly believed value, but M_F can increase substantially depending on T_{init} and κ. In particular, molecular clouds with lower metallicity yield a larger M_F inversely proportionally to metallicity. For $T_{init} = 10$ K, a cloud with $\kappa = 10^{-4}$ cm^2 g^{-1}, which is smaller by a hundred than the typical value for nearby molecular clouds, yields $M_F = 3.7\ M_{\odot}$.

The critical densities derived here would be helpful to numerical calculations in which either isothermal or a harder EOS (e.g., $\gamma = 5/3$) is chosen corresponding to whether $\rho < \rho_{crit}$ or $\rho > \rho_{crit}$.

In the present investigation we suppose that the opacity per unit mass is independent of the density (i.e., assuming LTE) and is homogeneous throughout the cloud. This assumption is justified when the cloud density is so high that interstellar dust is the dominant coolant, but it is inapplicable for lower densities where the cooling by molecular lines mainly contributes to the opacity. For the molecular line cooling, LTE is not satisfied in general and the assumption that the opacity per unit mass is homogeneous is violated because the line feature depends on the velocity structure of clouds. Our future works will be intended to generalize the present results to be applicable to the molecular or atomic line opacity.

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REFERENCES

Appenzellar, I., & Tscharnuter, W. 1974, A&A, 30, 423
Beckwith, S. V. W., Sargent, A. I., Chini, R. S., & Güsten, R. 1990, AJ, 99, 924
Boss, A. P. 1988, ApJ, 331, 370
Burkert, A., Bate, M. R., & Bodenheimer, P. 1997, MNRAS, 289, 497
Inutsuka, S., & Miyama, S. M. 1992, ApJ, 388, 392 (IM92)
———. 1997, ApJ, 480, 681 (IM97)
Kanjilal, T., & Basu, B. 1992, Ap&SS, 193, 17
Larson, R. B. 1969, MNRAS, 145, 271
Low, C., & Lynden-Bell, D. 1976, MNRAS, 176, 367
Masunaga, H., Miyama, S. M., & Inutsuka, S. 1998, ApJ, 495, 346 (MMI)
Miyama, S. M., Narita, S., & Hayashi, C. 1987a, Prog. Theor. Phys., 78, 1051
Miyama, S. M., Narita, S., & Hayashi, C.. 1987b, Prog. Theor. Phys., 78, 1273
Penston, M. V. 1969, MNRAS, 144, 425
Rees, M. F. 1976, MNRAS, 176, 483
Silk, J. 1977, ApJ, 214, 152
Tohline, J. E. 1980, ApJ, 239, 417
Truelove, J. K., Klein, R. I., Mckee, C. F., Holliman, J. H., II, Howell, L. H., & Greenough, J. A. 1997, ApJ, 489, L179
Truelove, J. K., Klein, R. I., Mckee, C. F., Holliman, J. H., II, Howell, L. H., Greenough, J. A., & Woods, D. T. 1998, ApJ, 495, 821
Winkler, K.-H. A., & Newman, M. J. 1980, ApJ, 238, 311